Assignment IX, Math 187

Dr. Holmes

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This problem set is due on Monday, July 19. All questions are in the area of coverage of Test III.

1. Show that for any integers a and $b \neq 0$, $a + b\sqrt{2}$ is irrational. You may use the information that $\sqrt{2}$ is irrational without proof. Hint: $a + b\sqrt{2} = \frac{c}{d}$ can be solved for $\sqrt{2}$.

2. Arrange these fractions in increasing order (show all work: you may use your calculator only to add and multiply natural numbers):

$$\frac{61}{99}, \frac{3}{5}, \frac{17}{30}, \frac{601}{999}, \frac{599}{1001}$$

3. Compute the square root of 5 to four decimal places using the "long division"-like procedure demonstrated in class.

   Use the recursion formula $x_{n+1} = \frac{x_n + \frac{7}{x}}{2}$ to estimate the square root of 5 to as many decimal places as your calculator can handle. Show all work (here you may use your calculator freely (and you need to!)).

4. In a bin of 35 children's blocks, 27 are rectangular, 10 are green, and 5 are neither rectangular nor green. How many green blocks that are not rectangular are in the bin?

5. Counting pairs.

   (a) A committee has ten members. It has a number of subcommittees. Each subcommittee has five members. Each member of the committee belongs to three subcommittees.

How many subcommittees are there? Give your explanation in detail. For additional credit, make an assignment of members of the committee to subcommittees that satisfies these conditions.

(b) A committee has ten members. It has a number of subcommittees. Each subcommittee has four members. Each member of the committee belongs to three subcommittees.

Is this story possible? Explain why or why not.

6. In a class of 21 modern language majors, 11 study French, 11 study German, and 9 study Russian. 5 study French and German. 4 study French and Russian. 3 study German and Russian. Every student in the class studies at least one of these three languages. How many students in the class study all three languages? Give a complete account of your reasoning; illustrating it with a diagram is also a good idea.

7. A small state has license plates consisting of three letters followed by three digits.

   How many license plates are possible?

   How many license plates are possible that contain the letter A? (Hint: it's easier to think about the license plates that do not contain the letter A).

   How many license plates are possible that contain the digit 8?

   How many license plates contain the letter A and the digit 8? (This might be harder, depending on how you approach it).