2.8)  b) \(32 \rightarrow 1, 2, 4, 8, 16, 32\)
32 has 6 divisors

c) \(2^n\), \(n\) is positive integer has \(\frac{n+1}{2}\) divisors.

\(10 \rightarrow 1, 2, 5, 10\)
10 has 4 divisors

E) \(100 \rightarrow 1, 2, 4, 5, 10, 20, 25, 50, 100\)
100 has 9 divisors

F) \(1,000,000 \rightarrow 1, 2, 4, 5, 8, 10, 16, 20, 25, 40, 50, 80, 100, 125, 200, 250, 400, 500, 800, 1000, 1250, 2000, 2500, 4000, 5000, 8000, 10000, 12500, 20000, 25000, 40000, 50000, 80000, 100000, 125000, 200000, 250000, 400000, 500000, 800000, 1000000\)
1000000 has 14 divisors

4) \(10^n\), where \(n\) is positive integer has \(\frac{n+1}{2}\) divisors. Why?

3.4) A) "If A, then B" is true if either B happens or if B doesn't happen then A doesn't happen either.
B) "If (not B), then (not A) is true if either B happens or if B doesn't happen then A doesn't happen either.

Both statements are false if A happens but not B.
Since both statements are true and false under the same conditions, they are logically invalid.

3.8) The shortest distance between two points is the length of the line segment that connects them.
4.1) Proposition: If $x$ and $y$ are odd integers, then $x + y$ is an even integer.

Proof: By definition of odd, there exists an integer $a$ where $x = 2a + 1$ and there exists an integer $b$ where $y = 2b + 1$.

Therefore,
$$x + y = (2a + 1) + (2b + 1) = 2a + 2b + 2 = 2(a + b + 1)$$

Since $a$ and $b$ are both integers, $a + b + 1$ must also be an integer and $2(a + b + 1)$ must be even according to the definition of even. Since $x + y = 2(a + b + 1)$, $x + y$ must be even.

4.4) Proposition: Let $x$ be an even integer and let $y$ be an odd integer. The product of $x \cdot y$ must be even.

Proof: By definition of even, there exists an integer $a$ where $x = 2a$.

By definition of odd, there exists an integer $b$ where $y = 2b + 1$.

Therefore,
$$x \cdot y = 2a \cdot (2b + 1) = 4ab + 2a$$

Since the product of two integers is an integer and the sum of two integers is an integer, $4ab + 2a$ is an integer.

Therefore, $2(ab + a)$ must be even by the definition of even. Since $x \cdot y = 2(ab + a)$, $x \cdot y$ must also be even.
Proposition: Let $a$, $b$, and $c$ be integers. If $a \mid b$, then $a \mid (bc)$.

Proof: - By definition of divisibility, there exists an integer $x$ where $b = a \cdot x$.
- Therefore $b \cdot c = (a \cdot x) \cdot c = a \cdot (x \cdot c)$.

Since $x$ and $c$ are both integers, $x \cdot c$ must also be an integer.
Let $x \cdot c = \text{integer } z$.
Since $b \cdot c = a \cdot (x \cdot c)$, $b \cdot c$ also equals $a \cdot z$.
By the definition of divisibility, $a \cdot z$ is divisible by $a$ since $a \cdot z = a \cdot z$.
Since $b \cdot c = a \cdot z$, $b \cdot c$ is divisible by $a$.

9.10) Proposition: Let $x$ be an integer. $x$ must be odd if $x + 1$ is even, and $x + 1$ must be even if $x$ is odd. ($\Rightarrow$)

Proof:
($\Rightarrow$) - Suppose $x$ is an odd integer. By the definition of odd, we have that there is an integer $a$ such that $x = 2a + 1$.
- Then $x + 1 = (2a + 1) + 1 = 2a + 2 = 2(a + 1)$.
- Since $a$ is an integer, we have that $a + 1$ is also an integer.
By the definition of even, we have that $x + 1$ is even.

($\Leftarrow$) - Suppose $x + 1$ is even. By the definition of even, we have that there is an integer $b$ such that $x + 1 = 2b$.
Therefore $x - 1 = 2b - 2$.
And $x - 1 = 2(b - 1)$.
and $x = 2(b - 1) + 1$. 

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Since \( b \) is an integer and \( 1 \) is also an integer, by subtraction \( b - 1 \) is also an integer. 

Let \( b - 1 = \text{integer } c \). 

We have \( x = 2c + 1 \). 

By definition, if odd, we have that \( x \) is odd. 

Very good work!