Please work in groups with no more than four people and complete this worksheet during class. Hand in one worksheet for each group.

1. Consider the function $f(x) = x^4 + 4x + 1$.
   
   (a) Use the Intermediate Value Theorem to show that $f(x) = 0$ has at least one real root.

   (b) Estimate an interval on which this function satisfies the hypotheses of Rolle’s Theorem (use your calculator). Then find all numbers $c$ in your interval that satisfy the conclusion of Rolle’s theorem.
(c) Graph $f(x)$ and illustrate all values of $c$ found in (b). From the graph of $f(x)$ estimate the number of real roots of $f(x)=0$ and label them on the graph as $a, b, d, e, ...$ (i.e. if you think there is one label it $a$, if two label them $a, b$, etc.).

(d) Assume that there are three roots of $f(x) = 0$ (even if in (c) you did not estimate that there are three roots) and denote them as $a, b, d$ with $a \leq b \leq d$. Show that the hypotheses of Rolle’s theorem are satisfied on $[a, b]$ and $[b, d]$. 
(e) State the conclusion of Rolle’s theorem on each interval \([a, b]\) and \([b, d]\).

(f) You should notice that something is wrong in (d). What is it? This means that our assumption that there are three roots is incorrect and there are \textit{at most} two real roots.
2. Show that the equation $x^4 + 4x + c = 0$ has at most two real roots.
   
   *Hint: You did almost all of the work in 2 (d)-(f)*
3. Just after you get on I84 from I184 you pass a car pulled over by the highway patrol. Feeling glad that it isn’t you, and relatively safe because you have a radar detector, you blast on by. One hour and ninety miles down the road, however, another patrolman pulls you over. You’re not worried, because your detector had warned you to slow down. Imagine your shock, however, when he proceeds to write you a ticket for going 90 miles per hour! When you protest, he tells you that the patrolman at I184 had radioed your position ahead. He then claims that since your average speed was ninety, the Mean Value Theorem says you must have been going ninety at least once in your journey. Explain why your speeding ticket is valid.