Names ____________________________

Please hand in one worksheet per group of 2-4 people.

The following will help you evaluate the integrals which result from partial fractions:

\[
\int \frac{1}{x^2 + a^2} \, dx = \frac{1}{a} \tan^{-1} \left( \frac{x}{a} \right) + C
\]

\[
\int \frac{f'(x)}{f(x)} \, dx = \ln |f(x)| + C
\]

I will guide you through the first exercise, and then give you subsequently harder problems.

1. You will solve

\[
\int \frac{2x^3 - 4x^2 - 5x + 3}{x^2 - 2x - 3} \, dx
\]

(a) Use long division to re-write the rational function

\[
\frac{2x^3 - 4x^2 - 5x + 3}{x^2 - 2x - 3}
\]

as a polynomial plus a proper fraction.
(b) Find $A$ and $B$ so that

$$\frac{x + 3}{x^2 - 2x - 3} = \frac{A}{x - 3} + \frac{B}{x + 1}.$$ 

Do this by multiplying both sides of the equation by $(x - 3)(x + 1)$, and cancel the $(x - 3)s$ and $(x + 1)s$ as much as possible. Find $A$ and $B$ by solving two equations; one equation involves the constants and the second involves all coefficients of terms involving $x$. Check your work by adding the two fractions together.
(c) Use your results to solve \[ \frac{2x^3 - 4x^2 - 5x + 3}{x^2 - 2x - 3} \, dx. \]

(d) Some of the terms in your answer involve logarithms. Combine those terms into a single \( \ln \) term.
2. \[
\int \frac{9x + 32}{x^2 + 6x + 8} \, dx
\]
\[ \int \frac{3x^3 - 2x - 2}{(1 + x)x^3} \, dx \]
4.

\[ \int \frac{2x^2 - 3x + 22}{(x - 4)(x^2 + 5)} \, dx \]