1. (11 points) Find the maximum and minimum values for

\[ f(x) = 1 - 3x + 5x^2 - x^3 \]

on the interval \([-2, 4]\)

2. (a) (5 points) State the Mean-Value Theorem.

(b) (6 points) Draw a picture of a nonlinear function which satisfies the hypotheses of the Mean Value Theorem and illustrate on this graph a geometric interpretation of the Mean Value Theorem.
3. (11 points) Sketch the graph of a function that satisfies all of the following conditions.
\[ f(0) = 0, \quad f(-1) = 1; \quad f'(-1) = 0 \]
\[ f''(x) > 0 \text{ on } (-\infty, -1) \]
\[ f'''(x) < 0 \text{ on } (-1, 0) \text{ and on } (0, \infty) \]
\[ f''(x) > 0 \text{ for } x > 0 \quad f' < 0 \text{ for } x < 0 \]
4. (11 points) Sketch a graph of the function $f$ defined below. To help, I have given you the simplified formulae for the first and second derivatives of $f$ as well as the formula of $f$ itself. Since you can plot $f$ using your calculators, the actual plot does not count very many points (not more than 2.) The points are awarded based upon your work.

\[
\begin{align*}
f(x) & = \frac{x + 2}{(1 - x)^{3/2}} \\
f'(x) & = -\frac{x + 8}{2(x - 1)^{5/2}} \\
f''(x) & = \frac{3(x + 14)}{4(x - 1)^{7/2}}
\end{align*}
\]
5. (11 points) If $f'(x) < 1$ for all $x \in [-4,4]$ and $f(-1) = 0$, is it possible for $f(2) = 4$? Why or why not?

6. Use the following graph of $f'$ to answer the following questions about $f$. (N.B. The graph of $f$ is not shown!!)

(a) (5 points) On what intervals is $f$ increasing and on what intervals is $f$ decreasing?

(b) (5 points) At what values of $x$ does $f$ have a local maximum and at what values of $x$ does $f$ have a local minimum.

(c) (5 points) On what intervals is the graph of $f$ concave up and on which intervals is the graph of $f$ concave down.

(d) (5 points) Assuming $f$ is continuous, and that $f(0) = 0$, sketch a possible graph of $f$. 
7. (10 points) Find the area of the largest rectangle that can be inscribed in the upper half of a circle of radius 6 if one side of the rectangle must be on a diameter of the circle.

8. (15 points) If \( f(1) = 2, \ f'(-1) = 1 \) and \( f''(x) = -2x + 4 \), what is the formula for \( f \)?