Find the derivative of each of the functions given in problems 1–6. Simplify your answers for problems 1–3.

1. \( f(x) = 2x^2 - 5x + 4 \)
2. \( f(x) = (x + 4)^3(x - 3)^2 \)
3. \( f(x) = \frac{x}{\sqrt{x^2 + 1}} \)
4. \( f(x) = \int_0^x \sqrt{1 + t^4} \, dt \)
5. \( f(x) = \frac{2x^2 \ln(\sin(x)) + 5}{\arctan(x) + 1} \)
6. \( f(x) = (x + (2x + (3x + 4)^5)^6)^7 \)

Evaluate the following limits (Problems 7–11):

7. \( \lim_{x \to 2} (x^2 + 5x - 2) \)
8. \( \lim_{x \to 2} \frac{x^2 - 6x + 8}{x - 2} \)
9. \( \lim_{x \to \infty} \left( \frac{x + 2}{x} \right)^x \)
10. \( \lim_{x \to 0} \sin(x) \ln(\tan(x)) \)
11. \( \lim_{x \to -\infty} (x + \sqrt{x^2 - 2x}) \)

Evaluate each of the integrals in problems 12–16.

12. \( \int (2x^2 - x + 4 - \frac{x}{1 + x^2}) \, dx \)
13. \( \int_0^2 (x\sqrt{4 - x^2}) \, dx \)
14. \( \int \frac{e^{\tan(x)}}{\cos^2(x)} \, dx \)
15. \( \int \frac{1}{x^2 + 2x + 5} \, dx \)
16. \( \int \frac{e^x + e^{-x}}{e^x - e^{-x}} \, dx \)

17. Find (and identify) the local extrema of the function \( f(x) = (x - 2)^2(x - 4)^3 \).
18. Give a calculus proof to show that among all rectangles of area 9 square feet, the square has the smallest perimeter.
19. State the $\epsilon$-$\delta$ definition of the limit as $x$ goes to $a$ of $f(x)$. 

20. State the Fundamental Theorem of Calculus.

21. Using the (limit) definition of the derivative, prove that the slope of the tangent line to the graph of $f(x) = x^2$ at the point $(3, 9)$ is 6.

22. Give an “$\epsilon$-$\delta$” proof of the fact that $\lim_{x \to 3}(4 - x) = 1$.

23. Sketch the graph of 

$$f(x) = \frac{x^2}{(x - 2)^2}.$$ 

24. If the length of a side of an equilateral triangle is increasing at a rate of 2 in./min, how fast is the area of the triangle increasing when the side is 7 inches long.

25. Find the area of the region between the graphs of $y = \cos(x)$ and $y = 2$ from $x = \pi$ to $x = 2\pi$. 
