1. Matrices $A$ and $B$ are $2 \times 3$, $C$ and $D$ are $3 \times 3$. Which of the following matrix operations are defined, and, for those that are, what is the size of the resulting matrix.

(a) $A + B$ (3)
(b) $AB$ (3)
(c) $AC$ (3)
(d) $AB^T + D$ (3)
(e) $A^TB$ (3)

2. Let $A$ and $B$ be $n \times n$ invertible matrices. Show that $AB$ is an invertible matrix and $(AB)^{-1} = B^{-1}A^{-1}$. (I know this is a theorem in the text. I want you to prove the theorem.) (10)

3. What is the matrix equation associated with the following system of equations:

\[
\begin{align*}
2x_1 &+ 3x_2 &- 3x_3 &= 5 \\
-x_1 &- 4x_2 &+ x_3 &= -2 \\
9x_1 & &- 3x_3 &= 0 \\
\end{align*}
\]

(10)

4. The following matrix is the reduced row echelon form of an augmented for a system of linear equations. Which variables are lead variables and which ones are free variables? What is the solution to the system?

\[
\begin{bmatrix}
1 & 2 & 0 & 0 & 3 \\
0 & 0 & 1 & 0 & 5 \\
0 & 0 & 0 & 1 & -2 \\
0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

(10)

5. If $A = \begin{bmatrix} 1 & 4 & -2 \\ 2 & 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 1 \\ 1 & 2 \end{bmatrix}$, find $BA$. (10)

6. Suppose $Ax = 0$ has a non trivial solution. Show that $Ax = b$ cannot have a unique solution. (10)

7. If $A = \begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 3 & 0 & -2 \\ 10 & 1 & 6 \\ 0 & 2 & 0 & -4 \end{bmatrix}$, find det($A$). (10)
8. Let $A = \begin{bmatrix} 1 & 0 & 0 & 1 \\ -1 & 1 & -1 & -2 \\ 0 & 2 & 1 & 0 \\ -2 & 0 & 1 & -1 \end{bmatrix}$

(a) Show $A^{-1} = \begin{bmatrix} -7 & -2 & 1 & -3 \\ 3 & 1 & 0 & 1 \\ -6 & -2 & 1 & -2 \\ 8 & 2 & -1 & 3 \end{bmatrix}$

(b) Solve the equation $A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 4 \\ 1 \end{bmatrix}$

9. Use row operations to solve

\begin{align*}
x_1 + x_2 + x_3 &= 6 \\
2x_1 - x_2 + 2x_3 &= 6 \\
3x_1 + 4x_2 - x_3 &= 8
\end{align*}